

## Proving a Continuous Function is Increasing on an Interval

Given a continuous function  $f(x)$  it is possible to prove whether or not it is increasing on a given interval  $[a, b]$ . This is accomplished by examining the first derivative of the function. If  $f'(x) > 0$  for all  $x \in [a, b]$  then  $f(x)$  is increasing on the interval  $[a, b]$ .

**Proof:**

$$\text{By definition } f'(x) = \frac{f(b) - f(a)}{b - a}$$

If the interval is  $[a, b]$  then also by definition  $b > a$

Thus the denominator  $(b - a) > 0$

Since  $f'(x)$  and  $(b - a)$  are both positive, then  $(f(b) - f(a))$  must also be positive. This only occurs when  $f(b) > f(a)$ .

Because  $f'(x) > 0$  for all  $x \in [a, b]$  we can replace  $a$  with any arbitrary value  $c$  such that  $c \in (a, b)$ . We can also replace  $b$  with any arbitrary value  $d$  such that  $d \in (c, b)$ . We then see that for any  $c$  and  $d$  that  $f(d) > f(c)$ . Thus  $f(x)$  is strictly increasing on the interval  $[a, b]$ .

**Example:**

Prove  $f(x) = x^2$  is increasing on the interval  $[2, 10]$

$$f'(x) = 2x$$

$f'(x) > 0$  for all  $x > 0$  thus  $f'(x) > 0$  for all  $x \in [2, 10]$

We know that  $f'(x) = \frac{f(b) - f(a)}{b - a}$  and  $f'(x) > 0$  for all  $x \in [2, 10]$ , therefore  $f(x)$  is strictly increasing on the interval  $[2, 10]$