

A Rational Multiplied By An Irrational Is Irrational

Let $a \in \mathbb{Q}$ and $b \in \mathbb{R}, b \notin \mathbb{Q}$

Then $a*b \in \mathbb{R}$ and $a*b \notin \mathbb{Q}$

Proof:

Proof by contradiction. Assume that $a*b \in \mathbb{Q}$

Then $a*b = \frac{m}{n} : m, n \in \mathbb{N}$

$$b = \frac{m}{n} * \frac{1}{a} \quad \text{Divide Each Side By } a$$

$$b = \frac{m}{na} \quad \text{Simplify}$$

This is a contradiction as $b \notin \mathbb{Q}$ and cannot take the form $\frac{m}{n}$